DETECTION OF TRENDS AND PERIODICITIES IN LONG CLIMATIC SERIES

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ABSTRACT

Numerous papers have been published on climatic change in hydrology and the atmospheric sciences in the recent past. The most commonly used methods for the study of trend, in particular, and seasonal effects are devised to find global patterns of these properties in a time series. By using moving averages, moving windows, filters and the like it is possible to see local effects of the trend or other component. Although these methods are advantageous they have some limitations because by fixing the length of the window estimates of a data trend are sensitive to the cut off time scale of the filter.
An alternative approach is to model the trend and periodic components with variances that are random variables. Such a representative time series model can be adopted advantageously, that is by making the method general and usable, through a Bayesian estimation procedure. Because there are usually computational problems to surmount, it seems that the only tractable way to implement this type of procedure is through a Markov chain Monte Carlo (MCMC) approach. In this way a random walk model is applied to model the state variables dynamically using available data. A time series model and Bayesian statistics are combined through a Markov chain procedure. A Gibbs sampling scheme is used in the Monte Carlo application.
Monthly series of riverflow, rainfall and temperature from northen Italy are used. In the trend components, they do not seem to show any general movement indicating a definitive rise or decay except that some late temperature rises are noted. There are strong similarities in behaviour between different localities. The periodicity components follow the annual cycle with variations from year to year. The random variations of periodicity are much stronger in the temperature series than in the other series. In general, the random component of periodicity has a much higher series of variances, than for trend.

1 INTRODUCTION

In recent years there has been an increase in publications on climatic change in hydrology and the atmospheric sciences. The most commonly used methods for the study of trend, in particular, and seasonal effects are devised to find global patterns of these properties in a time series. The general objective is to establish a mean trend line and an averaged periodicity component over the period of the record. Nevertheless, by using moving averages, moving windows, filters and the like it is possible to see local effects of the trend or other component. For, example, the Holt-Winters forecasting procedure, one of the earliest types, is based on simple exponential smoothing to model trend and periodicity; see recently Chatfield (1978).

New tools for smoothing climatic time series such as advanced spectral methods have been recently adopted. Although these methods are advantageous they have some limitations because by fixing the length of the window one cannot properly visualize temporal variations in properties such as trend in a time series. Long windows tend to cut off slow movements and short windows provide insufficient smoothing.

Of the sophisticated methods that have emerged during the past twenty years, wavelet analysis, devised for investigating localized variations of power (with reference to signals or other nonrandom behaviour) over a time series. There are several web sites and software is available with diverse climatic data sets such as sea surface temperatures, the Southern Oscillation Index and monsoon rainfalls. However, Baliunas et al. (1997), applied standard wavelet and adaptive wavelet transform algorithms to Central England temperatures but found that estimates of a data trend are sensitive to the cut off time scale of the filter.

An alternative approach, which does not depend on any form of filter, is to model trend and periodicity as non stationary components in a time series formulation. Without assuming that a time series is stationary or it is subject to particular form of nonstationarity, one can make the model general and usable. Because the statistical properties such as the variances of the trend and periodic components are unknown, these aspects should be incorporated in the algorithm. Such a representative time series model can be adopted advantageously through a Bayesian estimation procedure. Because there are usually computational problems to surmount in a direct application, it seems that the only tractable way to implement this type of procedure is through a Markov chain Monte Carlo approach, referred to by the acronym MCMC. In this way a random walk model is applied to model the state variables dynamically using available data. Here the term 'dynamic' refers to changes in a process with respect to time.

2 BAYES' THEOREM AND MCMC SAMPLING

Through the Bayesian framework one can combine initial assumptions about the parameters, such as trend and periodicity, of a model with observations to obtain the posterior distribution on the parameter space.

For example, consider a time series of a climatic variable such as temperature. This can be expressed in the classical additive form by

$$y_i = t_i + p_i + \eta_i \tag{1}$$

in which at time i, y_i represents the observed value of the variable, t_i and p_i are the parameters of trend and periodicity, respectively, and η_i is a random component.

One uses Bayes' theorem to update the current estimates of the parameters on the basis of new information. This can be summarized as follows. Let θ represent the unknown parameter space such as trend and periodicity in our model, the prior probability of which is denoted as $P(\theta)$ based on current knowledge of the process. After receipt of additional data y, the posterior distribution $P(\theta|y)$ is found from

$$P(\theta|y) = \frac{P(\theta)P(y|\theta)}{P(y)}$$

in which $P(y|\theta)$ is the likelihood function and $P(y)$ is a normalising quantity. In applications, however, problems usually arise because it is difficult to estimate the moments of the posterior distribution as shown here. Such computational difficulties can be overcome by means of Markov chain Monte Carlo (MCMC) sampling which is a simulation procedure of recent origin (see Gilks et al., 1996). The Metropolis – Hastings algorithm provides a general form of MCMC simulation developed by Hastings (1970) following Metropolis et al. (1953). This is an iterative procedure, some practicalities of which are described by Chib and Greenberg (1995). The idea is to construct a Markov chain whose stationary and ergodic distribution is the posterior distribution from Bayes' theorem (usually intractable by analytical means). However, repeated simulations are required to reach the desired states of stationarity and ergodicity.

In the hydrological literature, initial applications of the Metropolis – Hastings algorithm were limited to assessing parameter uncertainty in conceptual rainfall-runoff modelling (Kuczera and Parent, 1998; Bates and Campbell, 2001; and Marshall et al., 2004). It is implied in all cases that the model is correct and the source of error arises from the estimation of parameters. See also subsequent work of Efendiev et al. (2005), Reis Jr. and Stedinger (2005) and Renard et al. (2006).

A particular type of MCMC algorithms is the Gibbs sampler. This is devised to enable one to solve practical statistical problems. The advantages are its simplicity and convergence to the stationary and ergodic states. The Gibbs sampler has been used in medicine, archaeology, and many other fields (see, for example, Smith and Roberts (1993), Casella and George (1992)). However, some types of probability density functions or criteria or the fact that the model is not tractable enough, warrant the use of the wide and general class of Metropolis – Hastings algorithms. The basic concept of a Gibbs sampler can be visualized by considering a model represented by a vector of 3 parameters (A,B,C). Consider also an input *X*. One assumes that the initial state of the vector is $\left(A^0,B^0,C^0\right)$. In general, we sample A^i from the distribution $P\left(A \middle| B^{i-1},C^{i-1},X\right)$ then B^i from $P\left(B \middle| A^i,C^{i-1},X\right)$ and C^i from $P\left(C \middle| A^i,B^i,X\right)$ for $i=1,2,...,$ The conditionality in the distributions is the key feature. After the first iteration we transit to the state $\left(A^1,B^1,C^1\right)$ from the initial $\left(A^0,B^0,C^0\right)$. As one repeats the simulations over a long time period, a steady state is reached. Then the algorithm produces samples that correspond approximately to the distribution of $P(A,B,C|X)$. In this way, after receipt of data *X*, one can simulate the posterior distribution of the parameters as in Bayes' theorem.

Hydrological applications of the Gibbs sampling scheme have been made by Adamson et al. (1999) for simulating flood hydrographs in the Mekong River in Vientiane, Laos; Sansò and Guenni (1999) who simulated rainfalls in Venezuela; Perrault et al. (2000) for modelling annual energy inflows to two large hydropower dams in Quebec, Canada for change-point analysis in time series; and Onibon et al. (2004) who used the procedure to simulate Sahelian rainfields in West Africa.

3 THE WORKING ALGORITHM

The trend component of Eq.(1) can be written as

$$t_{i+1}=t_i+s_i\ , \tag{2}$$

using an auxiliary variable s_i. The uncertainty in the trend is modelled as

$$s_{i+1}=s_i+\omega_{1_i}\ , \tag{3}$$

in which ω_{1_i} is a 'latent' variable (in Bayesian terminology) that describes the random fluctuations in this component. With regard to the periodicity component, let us consider Eq(1) as a monthly time series and simplify this component so that it is modeled as a truncated Fourier series, in the usual way, but including only the fundamental frequency, $f=1/12$ (per month, corresponding to the annual cycle). This is closely representative for monthly mean temperatures and generally provides a good approximation for some other monthly climatic series. More about this follows. Accordingly, the periodicity component is modelled, following West and Harrison (1997) as

$$p_{i+1}=p_i\cos(2\pi f)+q_i\sin(2\pi f) \tag{4}$$

and

$$q_{i+1}=-p_i\sin(2\pi f)+q_i\cos(2\pi f)+\omega_{2_i}\ , \tag{5}$$

using an auxiliary variable q_i, and another 'latent' variable ω_{2_i} that describes the random fluctuations in this component. Thus the trend component is treated as a simple linear process whereas the periodicity component is a linear sum of sine and cosine curves. The advantage in this type of non stationary modelling is that one can incorporate differences in the cyclical structure and also changes in the mean level from year to year. The dynamic linear model of the system, also called the state space model, is then written as follows:

$$x_{i+1} = Fx_i + G\omega_i \tag{6}$$

$$y_i = Hx_i + \eta_i \tag{7}$$

where y_i represents the input data,

$$x_i = [t_i, s_i, p_i, q_i]^T ,$$

and

$$\omega_i = [\omega_{1_i}, \omega_{2_i}]^T .$$

Also,

$$H = [1 \quad 0 \quad 1 \quad 0], \tag{8}$$

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T , \tag{9}$$

and

$$F = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\pi/12) & \sin(2\pi/12) \\ 0 & 0 & -\sin(2\pi/12) & \cos(2\pi/12) \end{bmatrix} \tag{10}$$

such that Eq.(6) represents Eq.(2) to Eq.(5) and Eq.(7) represents Eq.(1). Because of the lack of prior knowledge we arbitrarily specify the initial conditions $x_0 = [t_0, s_0, p_0, q_0]$, of the variables in Eqs.(2) to (5) related to trend and periodicity.

Equations (6) and (7) are similar to the well known Kalman filter. However, we adopt a generalized approach by modelling the state variables as random walks in which the variances are treated as parameters within a Bayesian estimation framework, in the manner of West and Harrison (1997).

Accordingly, we write the stochastic dynamic model of Eqs. (6) and (7) in a static form which facilitates application of the Gibbs sampler. For the static representation we follow Magni and Bellazzi (2004):

$$y = Az + \eta \tag{11}$$

$$x = Bz \tag{12}$$

Here y represents n observations, as in Eq.(1), corresponding to which η signifies n measurement errors and x denotes the $2n$ values of trend t and periodicity p to be evaluated [Eqs.(2), (4)] and z is a variable incorporating the 4 initial conditions of t, s, p and q [Eqs.(2) to (5)] followed by the $2(n-1)$ 'latent' variables of Eqs. (3) and (5) of the trend and periodicity components.

Thus

$$x = [t_0 \quad p_0 \quad \dots \quad t_{n-1} \quad p_{n-1}]^T ,$$

$$y = [y_0 \quad \dots \quad y_{n-1}]^T$$

$$\eta = [\eta_0 \quad \ldots \quad \eta_{n-1}]^T$$

and

$$z = [t_0 \quad s_0 \quad p_0 \quad q_0 \quad \omega_{1_0} \quad \omega_{2_0} \quad \ldots \quad \omega_{1_{n-2}} \quad \omega_{2_{n-2}}]^T \ .$$

Also, to conform with Eqs. (6) to (10), the *A* and *B* matrices of constants in Eqs. (11) and (12) need to be redefined. A preliminary schematic diagram of the model of Eqs.(11) and (12) is given in Figure 1.

To implement the scheme, one needs to specify the probability distributions of the error terms. This also applies to the initial states. Let us assume that

$$P(\omega_{k_i}) = N(0, \sigma^2_{\omega_k}) \quad k = 1,2 \tag{13}$$

$$P(\eta_i) = N(0, \sigma^2_{\eta_i}) \tag{14}$$

$$P(x_0) = N\left(0, diag\left[\sigma^2_{x0_1} \quad \sigma^2_{x0_2} \quad \sigma^2_{x0_3} \quad \sigma^2_{x0_4}\right]\right) \tag{15}$$

where $N(.,.)$ denotes the multinormal distribution and $diag[\cdot\cdot]$ signifies a diagonal matrix.

In particular, $\sigma^2_{\omega_1}$ and $\sigma^2_{\omega_2}$ are treated as parameters to be estimated from the data within the Bayesian framework. We assume that the unknown variances in Eq.(13) pertaining to trend and periodicity have inverse gamma distributions which is expected as in the work of Gilks et al. (1996). For the prior state these distributions are vaguely defined, that is they are taken to be somewhat like uniform so that the final estimates do not have a large latent variable. Since the initial values of the *t*, *s*, *p* and *q* variables of Eq.(2) to (5) are assumed arbitrarily, as stated, their variances are taken sufficiently large so that the data will dominate the posterior distribution; see Eq.(15).

In order to update trend and periodicity through Bayes' theorem we compute the first two moments of the joint posterior probability

$$P(t, p, \sigma^2_{\omega_1}, \sigma^2_{\omega_2} | y) \tag{16}$$

where for *n* values of data *y*, $t = [t_0 \quad \ldots \quad t_{n-1}]^T$ and $p = [p_0 \quad \ldots \quad p_{n-1}]^T$ are the trend and periodicity components respectively. To implement the MCMC procedure through Gibbs sampling, we partition the random parameters into three groups or subsets: $\sigma^2_{\omega_1}$, $\sigma^2_{\omega_2}$ and *z*. It means that, the dependent structure shown in Fig. 1 should accommodate a scheme that extracts a sample in each iteration from each of the following three conditioned distributions (i.e. the distribution of a variable conditioned on the other variables and sample data, y) based on Eq. (11) and the previously stated strategy for Gibbs sampling:

$$P\left(\frac{1}{\sigma^2_{\omega_1}} \middle| \sigma^2_{\omega_2}, z, y\right) = \Gamma\left(n/2 + \gamma_1, \left(\omega_1^T \omega_1\right)/2 + \gamma_2\right) \tag{17}$$

$$P\left(\frac{1}{\sigma^2_{\omega_2}} \middle| \sigma^2_{\omega_1}, z, y\right) = \Gamma\left(n/2 + \gamma_3, \left(\omega_2^T \omega_2\right)/2 + \gamma_4\right) \tag{18}$$

$$P\left(z\middle|\sigma^2_{\omega_1},\sigma^2_{\omega_2},y\right)=N\left(D^{-1}A^T\sum\nolimits_{\eta-1}^{-1}y,D^{-1}\right) \tag{19}$$

with $D=A^T\sum_\eta^{-1}A+\sum_z^{-1}$

in which $\Gamma(.,.)$ is the gamma distribution and $N(.,.)$ is the multinormal distribution, σ^2_ω is a vector containing the sequence $\left\{\sigma^2_{\omega_1},\sigma^2_{\omega_2}\right\}$ repeated $(n-1)$ times, $\omega_i=\left[\omega_{i_0} \ \cdots \ \omega_{i_{n-2}}\right]^T$ with $i=1,2$ and $(\gamma_1,...,\gamma_4)$ pertain to the parameters of the *prior* gamma distributions of $1/\sigma^2_{\omega_1}$, $1/\sigma^2_{\omega_2}$. The sample data are given by y as defined previously.

Also, $\sum_\eta=diag\left(\left[\sigma^2_{\eta_0} \ \cdots \ \sigma^2_{\eta_{n-1}}\right]\right)$ and $\sum_z=diag\left(\left[\sigma^2_{x_{0_1}} \ \sigma^2_{x_{0_2}} \ \cdots \ \sigma^2_{x_{0_4}} \ \sigma^2_\omega\right]\right)$.

4 THE ITERATIVE PROCEDURE

The iterative procedure can be summarized as follows:

(1) The samples of $P\left(\sigma^2_{\omega_1},\sigma^2_{\omega_2},z\middle|y\right)$ are drawn iteratively by the Bayesian estimator and Gibbs sampler using Eqs.(17) to (19). This is in the main loop of the computer programme.

(2) We commence with prior values of the variances of ω_1 and ω_2 as in Eqs.(17) and (18) (regarding the parameters of their inverse gamma distributions).

(3) The vector z is simulated through the multinormal distribution of Eq.(19), based on Eq.(11), conditioned on sample data y and the two variances of ω_1 and ω_2. Recall that the variable z contains, except for the four initial conditions of t, s, p and q at the beginning, alternating values of ω_1 and ω_2. The vector z forms a row in the matrix Z, at each iteration.

(4) New values of ω_1 and ω_2 are abstracted from z and taken to Eqs.(17) and (18).

(5) Eqs.(17) and (18) provide the new variances of ω_1 and ω_2.

(6) The new variances are brought forward each time to Eq.(19).

(7) From these iterations *k* series (runs) are obtained for Z.

(8) Then the initial si series are discarded and the mean values of a z row vector are formed by averaging the $sr=k-si$ remaining series for each point in time. Values of si and sr are subject to investigation.

(9) We draw through Eq. (12) the sampling posterior distribution of the trend and periodicity components by a direct transformation of z to X.

5 RESULTS AND DISCUSSION

Figure 2 gives the monthly mean temperature at Chateauxdoex for the period 1901 to 2001 with the data grouped by years. Also shown are annual mean temperatures. As expected, the variability in the annual mean temperatures is seen to be much smaller than in the monthly data. The annual temperature data does not show a significant trend except for a rise after the third quarter of the past century, or thereabouts.

The Bayesian estimation and Gibbs sampling scheme is then applied to the monthly temperatures in Torino from 1901 to 2001, represented in Fig. 2. First, Fig.3 shows the trend component. This does not indicate any general movement. Except that there is a late rise during the fourth quarter of the last century as in Fig. 2. However, it shows a similar rise in the first quarter followed by a steady decline in the second and third quarters. Second, Fig. 4 shows the periodicity component for the some period. This has sharp irregular movements within periods of 12 months. The differences in amplitudes between years indicate the nonstationarity of the periodicity component.

It is found by considering differences in the averaged trend component from results by using sr [the remaining series in step 8 of section 4] and $sr/1.20$ for values of sr from 90, 100, 110, … ,250, that for sr = 200 an optimum is reached. Similarly differences in the averaged trend between sr series and discarded si series gave an optimum value for si = 50. Hence we decided to use values for the simulations in Gibbs sampling.

Figs. 5 and 6 give monthly total rainfalls in Asti for the period from 1881 to 2006 with the trend and periodicity components, respectively, via the Bayesian estimation and Gibbs sampling scheme. The trend component does not have any clear long-term movements. The periodicity component has oscillations that are much more regular than for the trend component with movements within the year due to its intrinsic nature; nonstationarity aspects are evident. We note how the extreme events in the initial part of the series influence both components more than in the central part. The 95% and 5% credibility limits are also shown. For this purpose we rank the x values at each point in time i through Eq. (12) (prior to averaging of z in step 8 of section 4) and obtain the rounded x values with 0.95 and 0.05 probabilities of exceedance.

Figs. 7 and 8 show monthly mean flows in the Po at Pontelagoscuro for the 24-year period from 1920 to 1991 with the trend and periodicity components respectively, via Bayesian estimation and Gibbs sampling. The trend component indicates local variations. These variations follow the general patterns in monthly flows. The periodicity component is further accentuated by the movements in the annual cycle and is much sharper than for the Milano rainfall data shown in Fig. 6. This seems to indicate an increase in the nonstationary behaviour but the scales are different. Both these figures do not show any definite movements. The 95% and 5% posterior credibility limits are also shown.

CONCLUSIONS

Seasonality is represented in the monthly data of this study. For some series Eqs.(4) to (10) need to be extended to incorporate a larger number of harmonics, than only the first harmonic applied here (Kottegoda et al., 2004). This also requires modifications or extensions to several equations from Eq. (4) to Eq. (19). Also, the assumption of normality made in Eqs. (13) to (15), as usually made in this type of general algorithm, requires further scrutiny although Figs. 2 and 3 do not seem to contradict the assumption. Alternatively, more advanced general simulation models may be used, see West and Harrison (1997), Carlin et al. (1992) and Carter and Kohn (1994).

Differently from other analyses of climatic time series, trend and periodicity are treated as random components by us. Elsewhere, smoothing and filtering seem to be the norm. We are motivated by considerations that such techniques may sometimes lead to biased results arising mainly from the width of the chosen window. That is, we have not been confined to a prior definition of the width of the moving window.

Three types of climatic time series are studied. The trend components do not seem to show any general movement indicating a definitive rise or decay, except that some late temperature rises are noted. No strong conclusions can be reached. There are similarities in behaviour between different types. However, the trend in the flow series is less oscillatory. The time varying periodicities seem to be less more pronounced in the rainfall series, but the scales are different.

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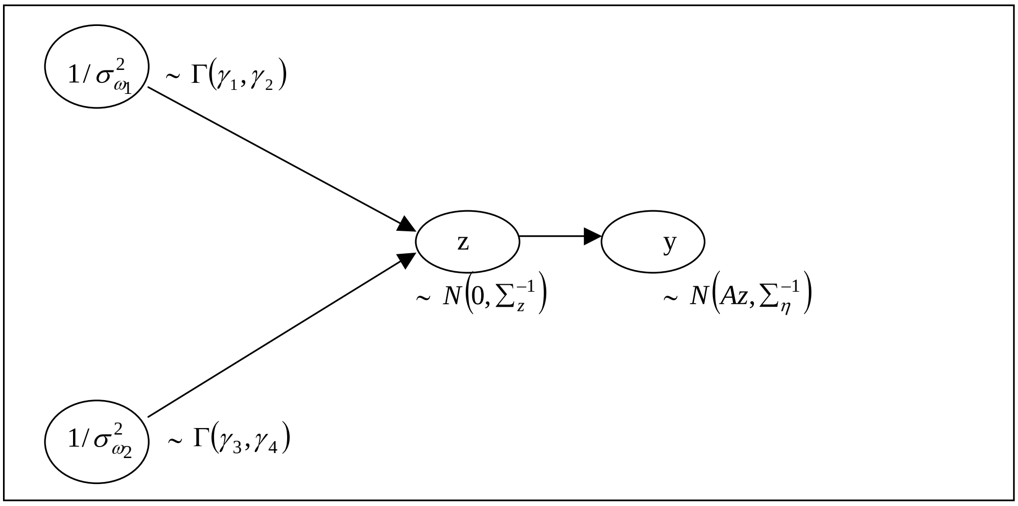

Figure 1. *Schematic diagram for implementing the Gibbs sampling scheme*

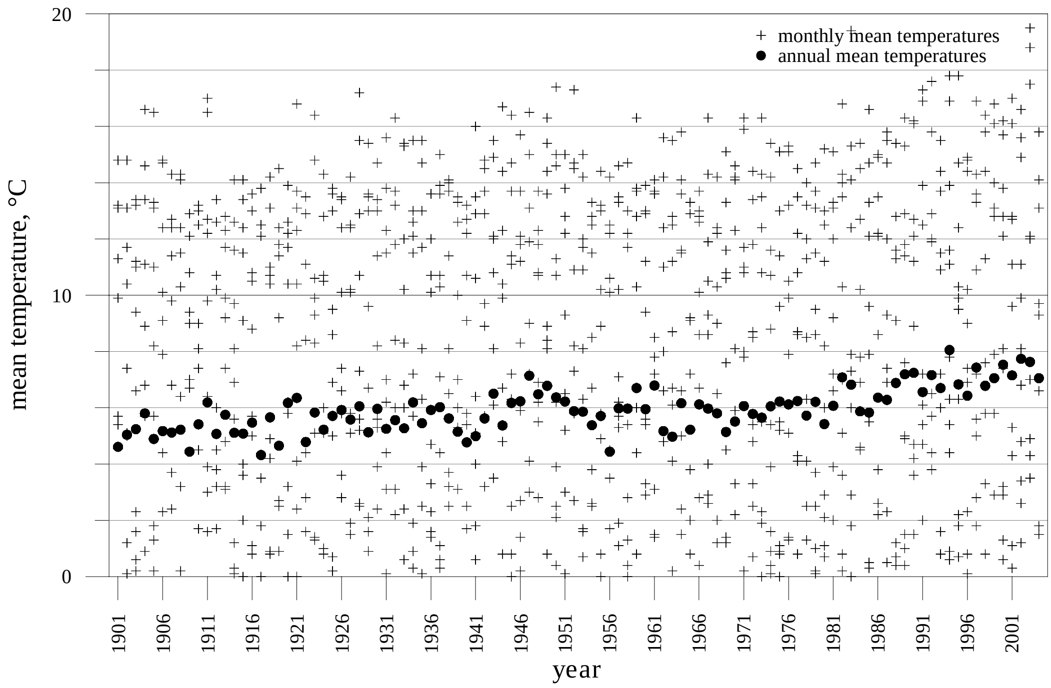

Figure 2. *Monthly mean temperature at Chateauxdoex for the period 1901 to 2004 - data grouped by years*

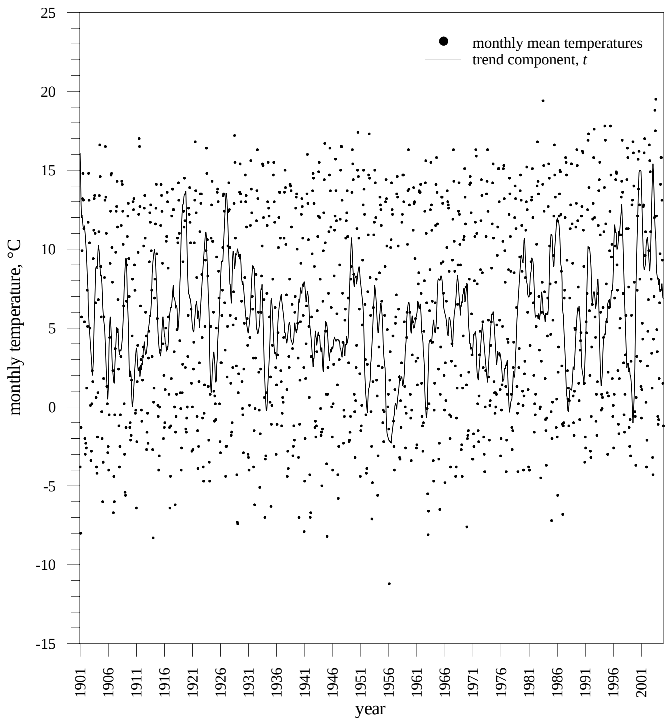

Figure 3. *Monthly mean temperature in Chateauxdoex for the period 1901 to 2004 with the trend component t from Gibbs sampling*

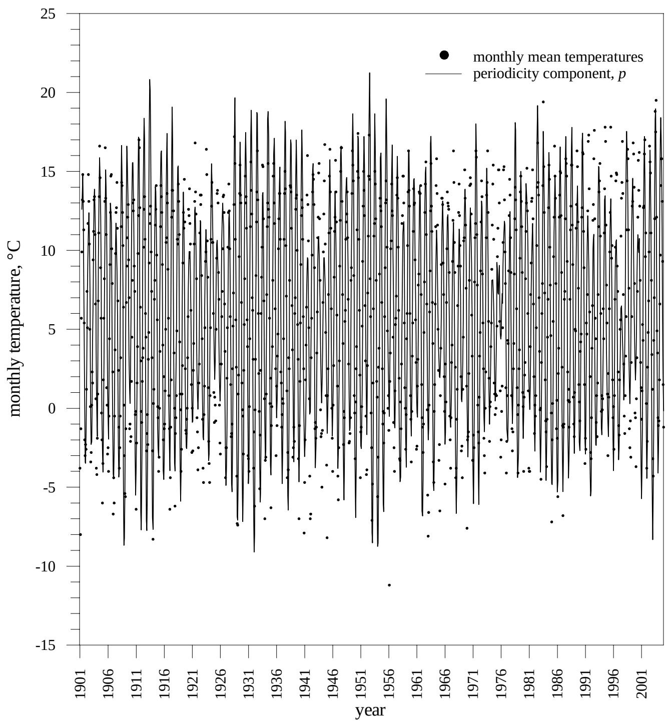

Figure. 4. *Monthly mean temperature in Chateauxdoex for the period 1901 to 2004 with the periodicity component p from Gibbs sampling*

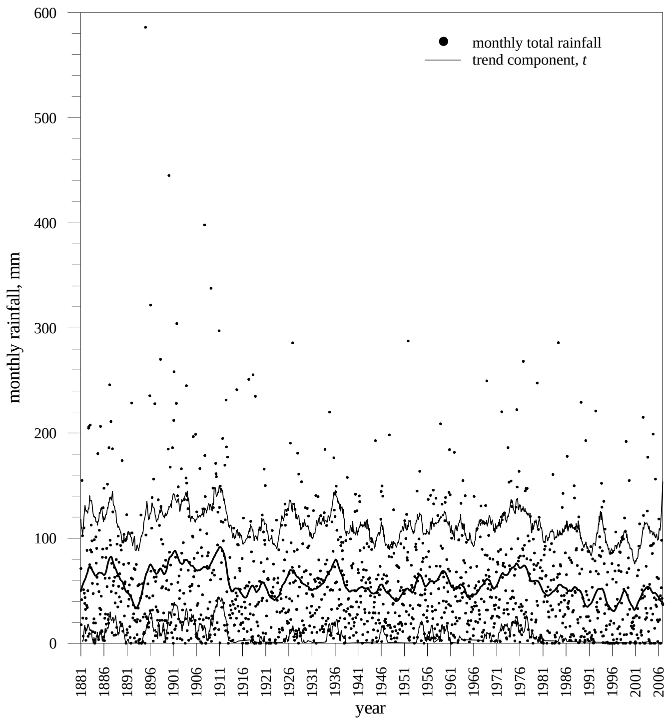

Figure 5. *Monthly total rainfall in Asti for the period 1881 to 2006 with the trend component t from Gibbs sampling*

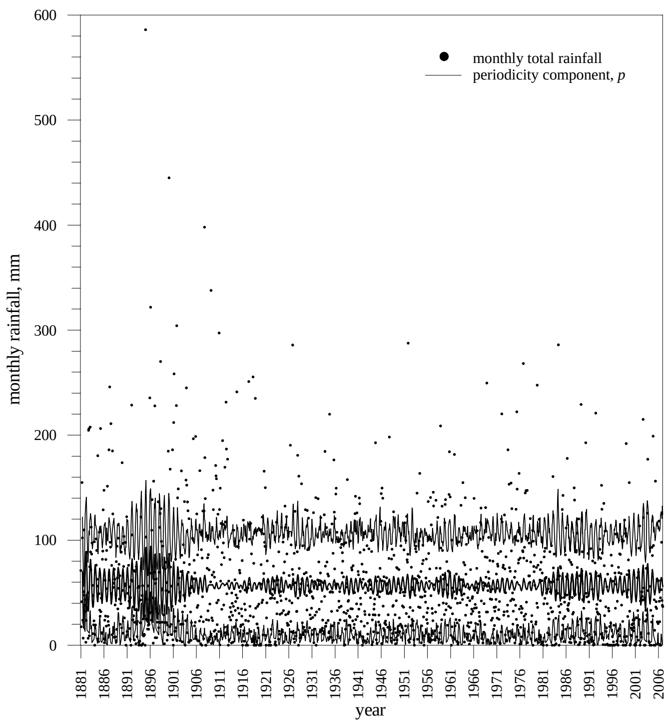

Figure 6. *Monthly total rainfall in Asti for the period 1881 to 2006 with the periodicity component p from Gibbs sampling*

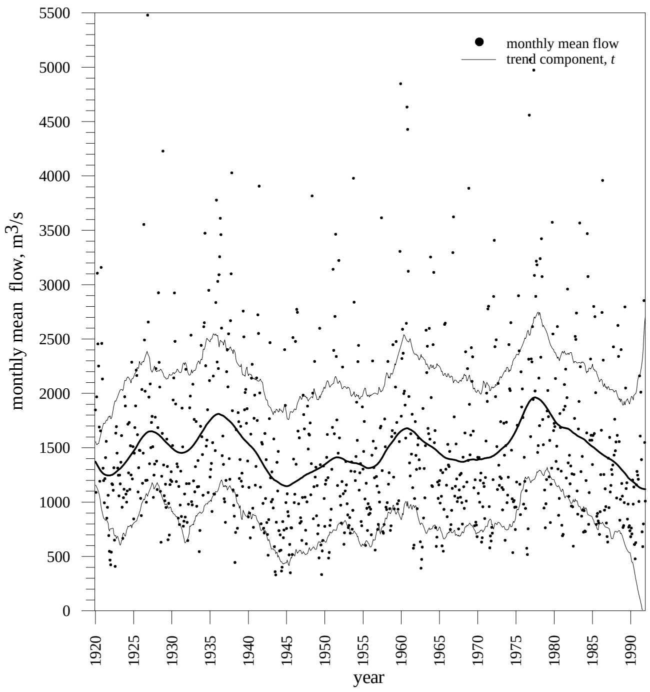

Figure 7. *Monthly mean flow in the Po at Pontelagoscuro for the period 1920 to 1991 with the trend component t from Gibbs sampling - Also shown are the 95% and 5% posterior credibility limits*

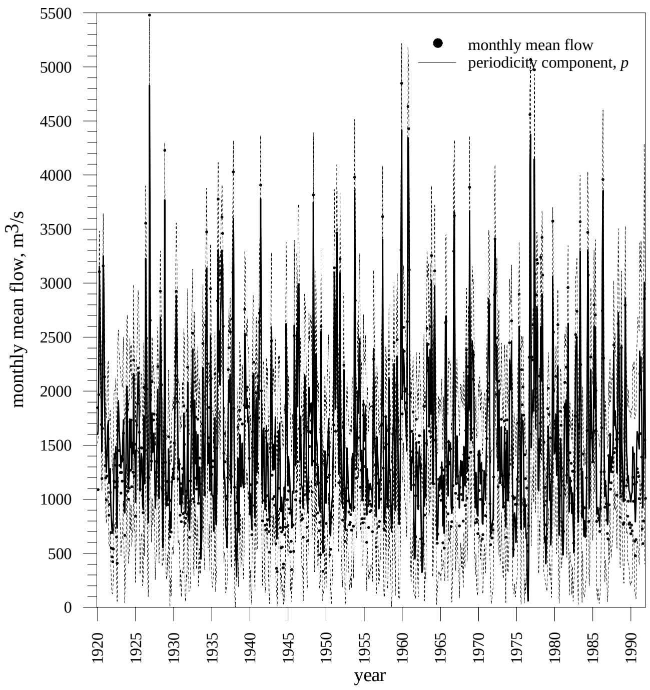

Figure 8. *Monthly mean flow in the Po at Pontelagoscuro for the period 1920 to 1991 with the periodicity component p from Gibbs sampling - Also shown are the 95% and 5% posterior credibility limits*